Games, graphs, and machines



September 20, 2024

A regex

Let *L* be the language on $\{0, 1\}$ consisting of *w* that contain an even number of 0s. Can you construct a regex that describes *L*?

o's appear in pairs () (01°011) V (10101*)* 人* separated by 15. () String made by concat is and Olimos String made of blocks 1-101-101-101-10 misses 1-1 so add a 1* at end.

An automaton

Let *M* be the language on $\{0, 1\}$ consisting of *w* that contain an even number of 1s. Last time, we constructed an autamoton that describes *L* (and similarly *M*). Construct automata that describe $L \cup M$ and $L \cap M$.



Given two languages L and M described by automatons A and B, construct automata that describe $L \cup M$ and $L \cap M$.

States = States of A × States of B

$$T_{Y}: (a, b) \xrightarrow{O} (a', b') \xrightarrow{a \circ a'} b \xrightarrow{a \cdot b'}$$

Start at (Starl A, Starl B)
Gauget : LUM : (a,b) if a cacept or
LOM : (a,b) if a accept and
b accept.

Regexes again?

Returning to L (even 0s) and M (even 1s), construct regexes that describe $L \cup M$ and $L \cap M$.

$$(0(m)^{*}0|1(00)^{*}1)^{*}$$

 $(0(m)^{*}0|1(00)^{*}1)^{*}$
 $M: ((10^{*}1)|0)^{*}$
 $L: ((010^{*}1)^{*})^{*}$

Regexes again?

Returning to L (even 0s) and M (even 1s), construct regexes that describe $L \cup M$ and $L \cap M$.

For $L \cap M$:

 $(0(11)^*0 \mid (1 \mid 0(11)^*10)(0(0 \mid 1(11)^*10))^*(1 \mid 01(11)^*0))^*$

Theorem Any language that can be described by a regex can also be described by an automaton, and vice-versa.

Regex -> automation

coming up



Searching for truth?

Let L = (Mathematical) statements that are true. Is there an automaton (or a Turing machine) that describes L?

NO (Godel) 1931

The true reason why [no one] has succeeded in finding an unsolvable problem is, in my opinion, that there is no unsolvable problem. [...] We must know. We shall know! – David Hilbert (1930)

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Last Q M, has matrix A M2 has matrix A